

A Non-Disjoint Group Shuffled Decoding for LDPC Codes

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Abstract—To reduce the implementation complexity of a belief propagation (BP) based low-density parity-check (LDPC) decoder, shuffled BP decoding schedules, which serialize the decoding process by dividing a complete parallel message-passing iteration into a sequence of sub-iterations, have been proposed. The so-called group horizontal shuffled BP algorithm partitions the check nodes of the code graph into groups to perform group-by-group message-passing decoding. This paper proposes a new grouping technique to accelerate the message-passing rate. Performance of the proposed algorithm is analyzed by a Gaussian approximation approach. Both analysis and numerical experiments verify that the new algorithm does yield a convergence rate faster than that of existing conventional or group shuffled BP decoder with the same computing complexity constraint.

I. INTRODUCTION

Low-density parity-check (LDPC) codes with belief propagation (BP) or so-called sum-product algorithm (SPA) based decoder can offer near-capacity performance. The SPA decoder, however, suffers from low convergence rate and high implementation complexity. To improve the rate of convergence and reduce implementation cost, serialized BP decoding algorithms which partition either the variable nodes (VNs) [1] or the check nodes (CNs) [2] of the corresponding bipartite graph into multiple groups were introduced. These two classes of serial SPA algorithms are called vertical and horizontal group shuffled BP decoding algorithms, respectively. More recent related works can be found in [3]–[6]. These practical alternatives use serial-parallel decoding schedules that perform sequential group-wise message-passings and have the advantage of obtaining more reliable extrinsic messages for subsequent decoding within an iteration.

In this paper, we focus on the horizontal group shuffled BP decoding algorithms as they provide more advantages in hardware implementation [1] [6]. For the sake of brevity, group shuffled BP (GSBP) stands for horizontal group shuffled BP (HGSBP) throughout this paper. For conventional GSBP schedules, the CNs are divided into a number of groups such that each CN belongs to just one group. A decoding iteration consists of several sub-iterations. Each sub-iteration updates in parallel the log-likelihood ratios (LLR) associated with the VNs connecting to the CNs in the same group. Hence within a sub-iteration, message-passing is performed on the bipartite subgraph that consists of the CNs of a group and all the VNs connecting to these CNs. Unlike conventional group shuffled

(GS) schedules which partition either VNs or CNs into disjoint groups, we propose a GS decoding schedule which divides CNs into non-disjoint CN groups. Such a CN grouping results in larger connectivity of consecutive subgraphs (CoCSG) associated with two neighboring CN groups, where the CoCSG, denoted by ℓ , refers to the the average number of VNs connecting the CNs of, say, the k th group and the VNs which are also linked to the CNs of the previous, i.e., $(k - 1)$ th, CN group. A larger CoCSG means more information will be forwarded from the previous sub-iteration and thus provides opportunities for improved decoding performance. We demonstrate by using both simulation and analysis that the proposed GSBP is indeed capable of offering significant performance gain and additional performance-complexity-decoding delay tradeoffs. Since our division on the CNs yields CN groups with a nonempty intersection for any two neighboring groups, we refer to the resulting decoding schedule as non-disjoint group-shuffled belief propagation (NDGSBP) in subsequent discourse.

To analyze the performance of iterative LDPC decoding algorithms in binary-input additive white Gaussian noise (BI-AWGN) channels, approaches such as density evolution (DE), Gaussian approximation (GA), and extrinsic information transfer (EXIT) charts have been proposed [7]–[11]. We adopt the GA approach [8] [11] as it requires just the tracking of the first two moments which are sufficient to completely characterize the probability densities. Moreover, if a consistency condition is met [11], we need to track only the means of related likelihood parameters.

The rest of this paper is organized as follows. In Section II, we explain the basic idea of the new grouping method, provide relevant parameter definitions and present the NDGSBP decoding algorithm. The corresponding GA-based performance analysis is given in Section III. Section IV provides numerical performance examples of the our algorithm, estimated by both computer simulations and analysis. Finally, concluding remarks are drawn in Section V.

II. NON-DISJOINT GROUP SHUFFLED BELIEF PROPAGATION ALGORITHM

A. Why GS decoding with non-disjoint groups?

Consider the decoding sub-iteration which performs VN-to-CN and then CN-to-VN message passing for the CNs of the

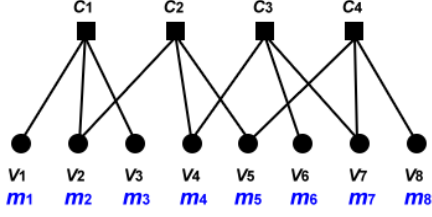


Fig. 1. The Tanner Graph of A Linear Block Code.

k th group and all connecting VNs. If (at least) one of the VNs is linked to some CNs in other (CN) groups which have been processed in the same decoding iteration before (i.e., whose group indices are smaller than k), then other connecting VNs which have no such links will benefit from receiving more newly updated messages. We use a simple linear code and its associating Tanner graph shown in Fig. 1, where there are four CNs $\{c_1, c_2, c_3, c_4\}$ and eight VNs $\{v_1, v_2, \dots, v_7, v_8\}$, to explain this effect. Let the messages the VNs carry be denoted by $m_1, m_2, \dots, m_7, m_8$. In a conventional BP decoding iteration, each VN receives the messages from its neighboring VNs which are linked through some VNs. For instance, v_4 and v_6 are updated by the messages $\{m_2, m_5, m_6, m_7\}$ and $\{m_4, m_7\}$, respectively. For the GSBP decoding with two CN groups $\{c_1, c_2\}$ and $\{c_3, c_4\}$, v_4 receives $\{m_2, m_5\}$ in the first sub-iteration and $\{m_2, m_5, m_6, m_7\}$ in the second sub-iteration while v_6 is updated by $\{m_2, m_4, m_5, m_7\}$ in which m_2 and m_5 are the messages forwarded by v_4 because of its connection to the second CN group and will help improving the convergence. Obviously, the amount of messages the CNs in the k group receive from VNs connected to CNs belonging to the j th group, $j < k$ depends on the code structure and the grouping of CNs. If we limit our attention to the case $j = k-1$, the single parameter ℓ defined in the introductory section can be used to quantify the average amount of messages received from the previous sub-iteration and the grouping should try to maximize this number.

To simplify our systematic non-disjoint grouping method, we assume identical group cardinality, N_G , and denote the number of CN groups by G so that $G \times N_G = M$ is the number of CNs. We define the overlapping ratio r as the ratio between the size of the intersection between two neighboring CN groups and G . Then, we have, $GN_G - (G-1)N_G r = M$.

We arbitrary select N_G CNs to form the first CN group. The k th ($k > 1$) group includes $r \cdot N_G$ CNs randomly chosen from the $(k-1)$ th group and $(1-r) \cdot N_G$ CNs from the CNs which do not belong to any of the earlier groups. Therefore, a CN does not necessarily belong to only one group anymore. As an illustration, we consider the grouping $(r, G, N_G) = (0.5, 3, 2)$ on the Tanner graph of Fig. 1 again. Let the first group be $\{c_1, c_2\}$, the second one be $\{c_2, c_3\}$ and the third one be $\{c_3, c_4\}$. In the first sub-iteration, v_2 and v_4 receive $\{m_1, m_3, m_4, m_5\}$ and $\{m_2, m_5\}$, respectively. v_4 and v_6 receive $\{m_1, m_2, m_3, m_5, m_6, m_7\}$ and $\{m_2, m_4, m_5, m_7\}$

in the second sub-iteration, in the final sub-iteration, v_6 will be updated by $\{m_1, m_2, m_3, m_4, m_5, m_7\}$. In short, for conventional BP, a VN can just collect information from VNs which are two-edge away in one iteration; for GSBP decoding, a VN has the opportunity to obtain the messages from four-edge-apart VNs; and for the proposed NDGSBP decoding algorithm, it is possible that a VN obtains the messages from VNs which are more than six-edge away if we select the overlapping ratio and CNs carefully. With fixed degree of parallelism N_G and CN number M , the larger r becomes, the longer the per-iteration delay is while the less the required iteration number becomes as a VN can update its LLR using information from more VNs. The product of the required iteration number and the per-iteration delay equals the total decoding delay to achieve a predetermined error rate performance. Section IV shows that the NDGSBP algorithm does give improved error rate performance for the same decoding delay.

B. Basic definitions and notations

A binary (N, K) LDPC code \mathcal{C} is a linear block code whose $M \times N$ parity check matrix $\mathbf{H} = [H_{mn}]$ has sparse nonzero elements. \mathbf{H} and thus \mathcal{C} can be viewed as a bipartite graph with N VNs corresponding to the encoded bits, and M CNs corresponding to the parity-check functions represented by the rows of \mathbf{H} . Given the above code parameters, the two parameters r and ℓ are related by $\ell \geq \frac{M}{N} \cdot N_G \cdot r$. More information is needed before an exact relation can be established. To track the statistical property variations of the message-passing sequence between VNs and CNs in an iterative decoding schedule, we also need to know the VN and CN degree-distribution polynomials $\lambda(x) = \sum_{i=2}^{d_v} \lambda_i x^{i-1}$ and $\rho(x) = \sum_{j=2}^{d_c} \rho_j x^{j-1}$, where λ_i and ρ_j denote the fraction of all edges connected to degree- i VNs and degree- j CNs, d_v and d_c denotes the maximum VN and CN degree.

Let $\mathcal{N}(m)$ be the set of variable nodes that participate in check node m and $\mathcal{M}(n)$ be the set of check nodes that are connected to variable node n in the code graph. $\mathcal{N}(m) \setminus n$ is defined as the set $\mathcal{N}(m)$ with the variable node n excluded while $\mathcal{M}(n) \setminus m$ is the set $\mathcal{M}(n)$ with the check node m excluded. Let $L_{n \rightarrow m}$ be the message sent from VN n to CN m and $L_{m \rightarrow n}$ be the message sent from CN m to VN n .

C. System model and decoding schedule

Assume a codeword $\mathbf{C} = (c_1, c_2, \dots, c_N)$ is BPSK-modulated and transmitted over an AWGN channel with noise variance σ^2 . Let $\mathbf{Y} = (y_1, y_2, \dots, y_N)$ be the corresponding received sequence and L_n be the log-likelihood ratio (LLR) of the variable node n with the initial value given by $L_n = \frac{2}{\sigma^2} y_n$.

Let \mathcal{G}_g be the g th CN group, $1 \leq g \leq G$ and \mathcal{U} be a set of CNs, l as the iteration counter and I_{Max} as the maximum number of iterations. We can then describe the NDGSBP algorithm as follows:

Initialization

Set $l = 1$, $\mathcal{U} = \{x | 1 \leq x \leq M\}$, and $\mathcal{G}_g = \emptyset$ for $1 \leq g \leq G$.

Step 1: Grouping check nodes

Collect N_G elements randomly from the set \mathcal{U} to form \mathcal{G}_1 , let $\mathcal{U} = \mathcal{U} \setminus \mathcal{G}_1$. Collect $N_G - N_G \cdot r$ element randomly from the set \mathcal{U} and $N_G \cdot r$ elements from \mathcal{G}_1 to create \mathcal{G}_2 . For $3 \leq g \leq G$, collect $N_G - N_G \cdot r$ element randomly from the set \mathcal{U} and $N_G \cdot r$ elements from $\mathcal{G}_{g-1} \setminus \mathcal{G}_{g-2}$ to create \mathcal{G}_g and let $\mathcal{U} = \mathcal{U} \setminus \mathcal{G}_g$.

Step 2: Message passing

For $1 \leq g \leq G$

- a) CN update: $\forall m \in \mathcal{G}_g, n \in \mathcal{N}(m)$

$$L_{m \rightarrow n} = 2 \tanh^{-1} \left(\prod_{n' \in \mathcal{N}(m) \setminus n} \tanh \left(\frac{1}{2} L_{n' \rightarrow m} \right) \right) \quad (1)$$

- b) VN update: $\forall n \in \bigcup_{m' \in \mathcal{G}_g} \mathcal{N}(m'), m \in \mathcal{M}(n)$

$$L_{n \rightarrow m} = L_n + \sum_{m' \in \mathcal{M}(n) \setminus m} L_{m' \rightarrow n} \quad (2)$$

Step 3: Total LLR computation

$\forall n, 1 \leq n \leq N$,

$$L_n^{total, (l)} = L_n + \sum_{m' \in \mathcal{N}(n)} L_{m' \rightarrow n} \quad (3)$$

Step 4: Hard decision and stopping criterion test

- a) Create $\mathbf{D}^{(l)} = [d_1^{(l)}, d_2^{(l)}, \dots, d_N^{(l)}]$ such that $d_n^{(l)} = 0$ if $L_n^{total, (l)} \geq 0$ and $d_n^{(l)} = 1$ if $L_n^{total, (l)} < 0$.
- b) If $\mathbf{D}^{(l)} \mathbf{H}^T = \mathbf{0}$ or I_{Max} is reached, stop decoding and output $\mathbf{D}^{(l)}$ as the decoded codeword. Otherwise, set $l = l + 1$ and $\mathcal{U} = \{x | 1 \leq x \leq M\}$, go to **Step 1**.

III. CONVERGENCE ANALYSIS

As can be seen from the above description of the proposed algorithm, the messages $L_{n \rightarrow m}$ and $L_{m \rightarrow n}$ are real random variables that depend on the received channel values y_n , the code structure and the decoding schedule. The GA approach assumes that they can be approximated by Gaussian random variables. With this approach, we need only to monitor the message means as the consistency condition holds in our case [7]. We further assume that the all-zero codeword $\mathbf{C} = (0, 0, \dots, 0)$, which is mapped into the BPSK modulated vector $\mathbf{X} = (1, 1, \dots, 1)$, is transmitted. The following analysis is based on the ideas of [8] and [11] with two distinct considerations. First, the analysis presented in [11] deals with vertical GSBP while we are dealing with horizontal GSBP. Second, the intersection among groups can be nonempty in our schedule. For GSBP decoding, we divide CNs into two types, one is updated CNs and the other is non-updated CNs. As depicted in Fig.2. To analyze the effect of nonempty intersections, we divide CNs into four classes in a given, say the g th sub-iteration of the l th iteration. Class-**a** includes the CNs that will be updated at the g' th ($g' > g$) sub-iteration, Class-**b** includes the CNs which are also members of the previous ($g - 1$)th group, Class-**c** contains the CNs which are

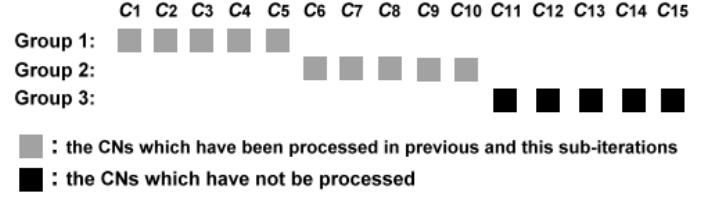


Fig. 2. A example for GSBP after two sub-iterations.

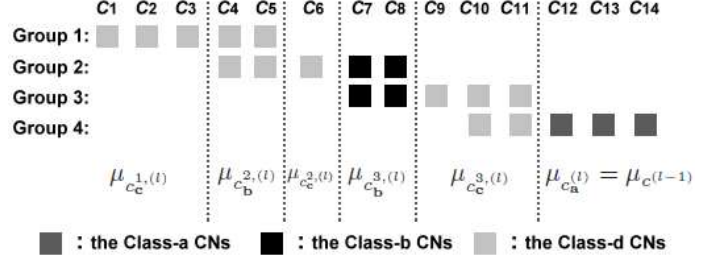


Fig. 3. A example for NDGSBP after three sub-iterations when $r < 0.5$.

not members of the previous ($g - 1$)th group and the Class-**d** are all CNs exclude Class-**a** and Class-**b**. Fig.3 and Fig.4 depict the situations after three sub-iterations for overlapping ratio $r < 0.5$ and $0.5 \leq r \leq 1$ respectively.

We now track the average values of all updated parameters at the l th iteration for the proposed NDGSBP algorithm. We first define $\mu_{c_x^{g, (l)}}$ as the mean of the message sent by a Class-**x** CN, that is, $\mu_{c_x^{g, (l)}} = E\{L_{m \rightarrow n}^{g, (l)}\}$, where m belong to Class-**x** CNs, n is a VN connecting to m in the g th sub-iteration of the l th iteration. We start with the VN update equation. Consider the degree- i VN n which is connected to p Class-**d** CNs, q Class-**b** CNs and $i - p - q$ Class-**a** CNs. For the g th sub-iteration of the l th iteration, we have, for $g = 1$,

$$\mu_{v_{i, p, q}}^{(l)} = \mu_0 + p\mu_{c_d^{(l)}} + q\mu_{c_b^{g, (l)}} \quad (4)$$

$$+ (i - p - q - 1)\mu_{c_a^{(l)}} \\ = \mu_0 + p\mu_{c_d^{(l)}} + q\mu_{c_b^{g, (l)}} + (i - p - q - 1)\mu_{c^{(l-1)}} \quad (5)$$

where $\mu_{c_d^{(l)}} = \mu_{c_c^{1, (l)}}$ and $\mu_0 \triangleq E\{L_n\} = E\{\frac{2y_n}{\sigma^2}\}$ is the mean

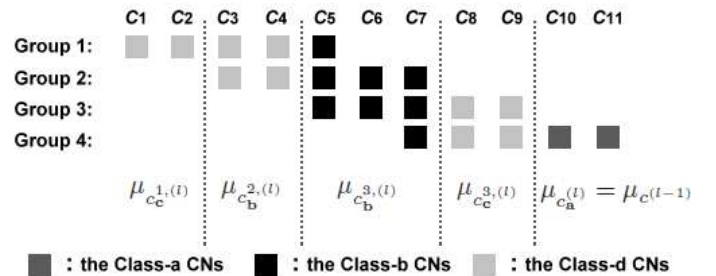


Fig. 4. A example for NDGSBP after three sub-iterations when $0.5 \leq r \leq 1$.

of the channel value. For $g > 1$, we obtain

$$\mu_{c_d^{(l)}} = \frac{1}{g} \left(\mu_{c_c^{1,(l)}} + \mu_{c_c^{g,(l)}} + \sum_{g'=2}^{g-1} \left(\frac{r}{1-r} \mu_{c_b^{g',(l)}} + \frac{1-2r}{1-r} \mu_{c_c^{g',(l)}} \right) \right), \quad (6)$$

for $r < 0.5$ and

$$\mu_{c_d^{(l)}} = \frac{1}{g} \left(\mu_{c_c^{1,(l)}} + \mu_{c_c^{g,(l)}} + \sum_{g'=2}^{g-1} \mu_{c_b^{g',(l)}} \right), \quad (7)$$

for $0.5 \leq r \leq 1$.

When the CNs in the g -th group are processed in l -th iteration, the mean of message for degree- i VNs $\mu_{v_i^{(l)}}$ can be obtained by accumulating all possible values of $\mu_{v_{i,p,q}^{(l)}}$ with their corresponding coefficients $\omega(i, p, q)$:

$$\mu_{v_i^{(l)}} = \sum_{p=0}^{i-1} \sum_{q=0}^{i-1-p} \omega(i, p, q) \cdot \mu_{v_{i,p,q}^{(l)}}, \quad (8)$$

where $\omega(i, p, q)$ is the proportion of degree- i VNs which have p neighboring Class-**d** CNs, q neighboring Class-**b** CNs in all degree- i CNs. Thus $\omega(i, p, q)$ is given by

$$\omega(i, p, q) = \begin{cases} \binom{i-1}{p} x^p (1-x)^{i-1-p}, & g = 1 \\ \binom{i-1}{p} \binom{i-1-p}{q} y^p z^q (1-y-z)^{i-1-p-q}, & g \neq 1 \end{cases} \quad (9)$$

where x is the fraction of Class-**d** CNs for $g = 1$, y is the fraction of Class-**d** CNs and z is the fraction of Class-**b** CNs.

Thus

$$x = \frac{1}{G - (G-1)r}, \quad (10)$$

$$y = \frac{g(1-r)}{G - (G-1)r}, \quad (11)$$

$$z = \frac{r}{G - (G-1)r}. \quad (12)$$

From Class-**c** CNs updating formula, we can obtain

$$E \left\{ \tanh \left(\frac{c_{c,j}^{g,(l)}}{2} \right) \right\} = \left[E \left\{ \tanh \left(\frac{v_i^{(l)}}{2} \right) \right\} \right]^{j-1}. \quad (13)$$

Under the Gaussian approximation and for $\mu \geq 0$, define

$$\Phi(\mu) \triangleq 1 - \frac{1}{\sqrt{4\pi\mu}} \int_{-\infty}^{\infty} \tanh\left(\frac{\tau}{2}\right) \exp \left[\frac{-(\tau-\mu)^2}{4\mu} \right] d\tau, \quad (14)$$

and (13) can be rewritten as

$$\mu_{c_{c,j}^{g,(l)}} = \Phi^{-1} \left(1 - \left(1 - \sum_{i=2}^{d_v} \lambda_i \Phi \left(\mu_{v_i^{(l)}} \right) \right)^{j-1} \right). \quad (15)$$

If we average over all CN degree j , we have

$$\mu_{c_c^{g,(l)}} = \sum_{j=2}^{d_c} \rho_j \cdot \mu_{c_{c,j}^{g,(l)}}. \quad (16)$$

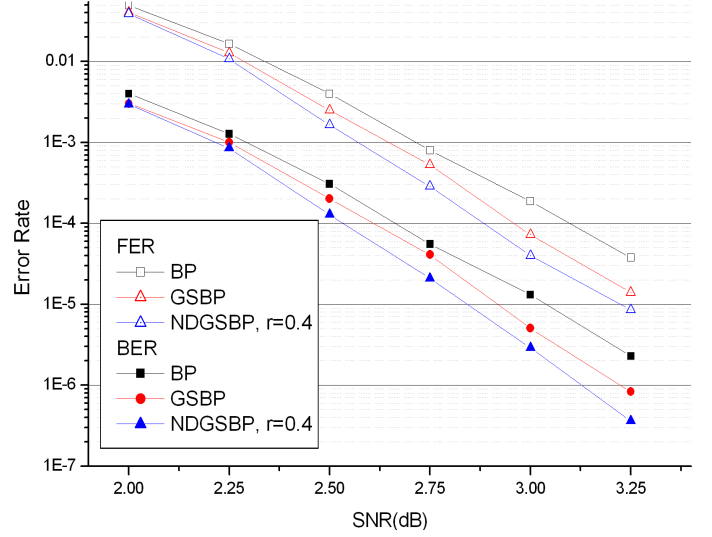


Fig. 5. BER and FER performance of Mackay's (504,252) regular LDPC code with $d_c = 6$ and $d_v = 3$ using the decoding algorithms: NDGSBP, GSBP for $G = 12$ and standard BP.

The computation of the mean of message send from a Class-**b** CN $\mu_{c_b^{g,(l)}}$ is replace $\mu_{v_i^{(l)}}$ with $\mu_{v_i^{(l)'}}$ in (15) where $\mu_{v_i^{(l)'}}$ is mean of message send from a previous group overlapping VN. And $\mu_{v_i^{(l)'}}$ is got by let p at least 1 in (8) and (9) for $g \neq 1$.

After l iterations, the mean of the message passed from a CN $\mu_{c^{(l)}}$ is

$$\mu_{c^{(l)}} = \frac{r}{G - (G-1)r} \mu_{c_b^{G,(l)}} + \frac{G - Gr}{G - (G-1)r} \mu_{c_c^{G,(l)}}. \quad (17)$$

If $\mu_{c^{(l)}} \rightarrow \infty$, the connecting VNs achieve error free performance.

IV. NUMERICAL RESULTS

Fig. 5 depicts the FER and BER performance of Mackay's (504,252) regular LDPC code with $d_c = 6$, $d_v = 3$ using the standard BP algorithm, the GSBP algorithm ($G = 12$) and the proposed NDGSBP algorithm ($G = 12$, overlapping ratio $r = 0.4$). On the other hand, in Fig. 6 we show the FER and BER performance of Mackay's (816,544) regular LDPC code with $d_c = 6$ and $d_v = 4$ using the standard BP algorithm, the GSBP algorithm ($G = 16$) and the proposed NDGSBP algorithm ($G = 16$, overlapping ratio $r = 0.4$).

The simulation results reported in this section assume $I_{Max} = 1000$ for the GSBP and BP algorithms. To have fair comparison, we assume the system parameter values that result in the same or similar computation complexity for all algorithms. For example, to decode the (504,252) LDPC code using the NDGSBP decoder with $G = 12$ and $r = 0.4$ imply that $N_G = 34$ and it is allowed to have at most $\frac{m \cdot I_{Max}}{m + (G-1) \times N_G \cdot r} = \frac{252 \cdot 1000}{252 + 11 \cdot 34 \cdot 0.4} \approx 627$ decoding iterations.

Fig. 5 indicates that at the BER 10^{-5} , the NDGSBP decoder is about 0.2dB better than the standard BP decoder, and achieves about 0.08dB decoding gain with respect to the GSBP decoder. Fig.6 also verify that the performance of the

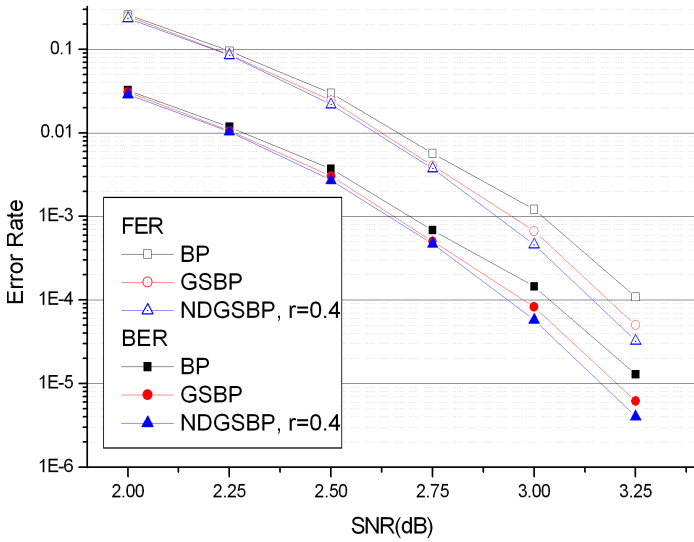


Fig. 6. BER and FER performance of Mackay's (816,544) regular LDPC code with $d_c = 6$ and $d_v = 4$ using the decoding algorithms: NDGSBP, GSBP for $G = 16$ and standard BP.

TABLE I

NUMBER OF DECODING ITERATIONS REQUIRED TO ACHIEVE ERROR-FREE PERFORMANCE FOR THE BP, GSBP AND NDGSBP ($r = 0.4$) DECODERS IN A BINARY-INPUT AWGN CHANNEL.

d_v	d_c	R	$(E_b/N_0)_{GA}$	BP	GSBP			NDGSBP		
					$G = 4$	12	36	$G = 4$	12	36
3	6	1/2	1.163	422	293	262	251	240	208	196
d_v	d_c	R	$(E_b/N_0)_{GA}$	BP	GSBP			NDGSBP		
					$G = 4$	16	34	$G = 4$	16	34
4	6	1/3	1.730	632	438	386	376	368	324	317

NDGSBP algorithm is superior to the BP and GSBP algorithm for the (816,544) LDPC code.

We use the GA approach outlined in Section III to analyze the performance of the NDGSBP, BP and GSBP decoders. Given the code rate and degree distribution of LDPC codes, the thresholds estimated by the GA approach for BP, GSBP and NDGSBP decoding are the same. In Table I, we list the number of iterations for error free performance at SNR equals threshold. We examine the NDGSBP performance in decoding two ensemble LDPC codes using the same overlapping ratio $r = 0.4$ but different group number G . The table shows the NDGSBP decoder consistently outperforms the other two decoders in convergence rate.

V. CONCLUSIONS

In this paper, we propose a new group shuffled BP decoding scheduling method to improve the performance of LDPC codes. Our scheme enhances the connectivity of the code graph by having overlapped CNs in neighboring CN groups. The enhanced connectivity allow more each VN (or CN) to obtain related information from more VNs (or CNs) within a decoding iteration, accelerating the message-passing rate and thus the convergence speed.

The GA approach is used to track the first-order statistical information flow of the proposed NDGSBP algorithm. The GA analysis verifies that the NDGSBP decoder does give faster convergence performance with respect to that of the GSBP and BP decoders. Numerical results also demonstrate that, with the same decoding computation complexity, the new algorithm yields BER and FER performance better than that of the conventional BP and GSBP decoders.

In this work, the VN order in grouping is arbitrary and the non-disjoint parts are randomly selected from the available CNs. A proper VN ordering and overlapping VN selection that take the code structure into account will certainly give better performance. The optimal decoding schedule and parameters (r, ℓ) remain to be found and some analytic performance metrics may be needed in our search of the desired solution.

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